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Optimal Takeoff of a Helicopter for Category A V/STOL Operations

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Two applications of a nonlinear optimal control theory are used to calculate the optimal control procedures for a helicopter following one engine failure during takeoff. One application is concerning the problem of optimization of the takeoff procedure for category A short-takeoff-and-landing (STOL) operation so that the required heliport size is minimized. The results show that the required takeoff distance using the normal takeoff procedure can be reduced by 30–60% if the takeoff path and the critical decision point are specifically optimized for a given set of the operating conditions, e.g., takeoff weight, ambient conditions, and heliport configuration. The second application concerns the problem of evaluation of the takeoff performance for category A vertical-takeoff-and-landing (VTOL) operation. The calculated maximum weight for the normal takeoff path shows good agreement with the certificated takeoff weight. Additionally, optimization of the takeoff path is shown to allow increasing the payload by 10%.

Nomenclature

H = height above takeoff surface
h_d = density altitude
m = helicopter mass
q = pitch rate
t = time from engine failure
u = horizontal flight speed
u_{max} = safety limit of horizontal flight speed at touchdown
V = flight speed
V_{TOSS} = takeoff safety speed
V_Y = best rate-of-climb speed
w = rate of descent
w_{max} = safety limit of rate of descent at touchdown
x = horizontal flight distance following engine failure
z = height loss following engine failure
γ = takeoff slope
θ_0 = collective pitch of main rotor blade
θ_s = cyclic pitch of main rotor blade
Θ = pitch attitude
Ω = rotor rotational speed

Subscript
f = final value

Introduction

TRANSPORT category helicopters are certificated as either category A or B.[1,2] Category A helicopters may be flown over areas where no emergency landing sites are available, and therefore regulations require that these helicopters be capable of being flown in the event of one engine failure once the critical decision point (CDP) on the takeoff path has been passed as shown in Fig. 1 [continued takeoff (CTO)]. If an engine fails before reaching the CDP, the helicopter is required to be capable of landing safely [rejected takeoff (RTO)].

The CDP has been historically defined by a fixed combination of height and velocity parameters which assure the safety of the CTO over the entire operating envelope. Such a CDP, however, requires a long RTO distance and has resulted in the necessity for a large heliport. Recently, variable CDP velocity was applied to the certification of the Sikorsky S-76B helicopter as STOL category A,[3] where it was demonstrated that a variable CDP velocity offers a distinct advantage in flexibility with respect to optimization of payload and field length. One objective of this study is, therefore, to calculate the optimal CDP velocity and the optimal takeoff path for category A STOL operation.

Helicopters possess vertical takeoff capability from a confined area as shown in Fig. 1. For category A VTOL operation, however, the takeoff weight is significantly limited because no unsafe region is allowed in the *H-V* diagram for the failure of one engine to assure the safety of an RTO. As a result, it is not practical to use commercial helicopters in VTOL operations, and consequently they are normally operated as STOL aircraft. Another objective of this study is to examine the possibility of expanding the current weight limitation for category A VTOL operation by applying an optimization theory.

To date, the maximum takeoff weight and the minimum takeoff distance for category A operation have been established by flight tests for certification. In order to reduce the cost, time, and risk involved in these flight tests, much research has been directed at theoretically predicting these operating limitations. Jepson[4] proposed a method to calculate the takeoff and landing characteristics following engine failure and the associated heliport size requirements. Cerbe and Reichert[5] applied a numerical optimization procedure to the quasistationary simulation model on the basis of power-

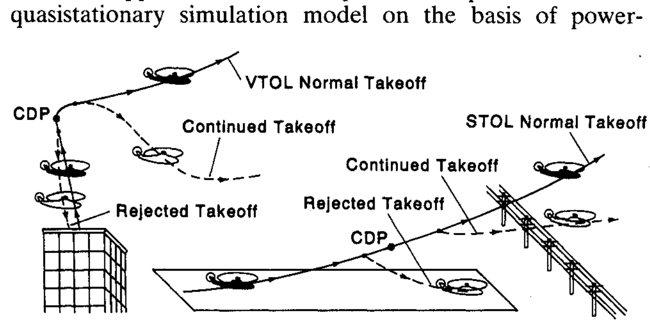

Fig. 1 Takeoff procedures for category A V/STOL operations.

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required data fields. Stevens and Vodegel[6] utilized a computer simulation based on the energy method for the certification of the S-76B helicopter as VTOL category A. The present authors developed a method to predict the *H-V* diagram of a single-engine helicopter using a nonlinear optimal control theory in conjunction with improved dynamic and aerodynamic helicopter models.[7] This method was also applied to the one-engine-inoperative (OEI) flight performance evaluation of a twin-engine helicopter.[8] Here, this method was extended to optimize the takeoff procedures for category A V/STOL operations.

Formulation of the Optimal Control Problems

The dynamic/aerodynamic helicopter models and formulation of the nonlinear optimal control problems are referred to in Refs. 7 and 8, with the performance index and boundary conditions being extended to include takeoff path optimization.

Dynamic/Aerodynamic Helicopter Models

The equations of motion are described assuming a rigid-body dynamic model having longitudinal three DOF. The state variables are horizontal flight distance x, height loss following engine failure z, horizontal flight speed u, rate of descent w, pitch attitude Θ, pitch rate q, and rotor rotational speed Ω. The control variables are the collective pitch θ_0 and longitudinal cyclic pitch θ_s of the main rotor. The external forces taken into consideration are the thrust, torque, *H*-force, and longitudinal hub moment of the main rotor, drag of the fuselage, lift of the horizontal stabilizer, etc. The available torque of the remaining engine is given as a function of the rotor rotational speed representing an engine governor model,[8] whereas the maximum torque in turn depends on the pressure altitude and air temperature. The aerodynamic performance of the main rotor is calculated by using a combination of the blade element theory and momentum theory which are modified to take into account the blade root stall during descent and the increased induced flow due to the vortex ring state. The following inequality constraints are included: collective pitch range, cyclic pitch range, pitch attitude range, rotor speed range, maximum load factor, and maximum effective angle of attack of the blade section at 75% span.

Performance Index and Boundary Conditions

The following optimization problems are formulated for five cases: 1a) minimizing the RTO distance for STOL operation; 1b) minimizing the CTO distance for STOL operation; 2a) maximizing the takeoff weight for VTOL operation; 2b) minimizing the height loss during the CTO for VTOL operation; and 2c) minimizing the touchdown speed during the RTO for VTOL operation. Table 1 summarizes the performance index and the terminal conditions for each case. Details of the corresponding equations are discussed in the following sections.

The initial conditions are established by the state variables 1 s after engine failure according to regulations[2] so that the normal pilot reaction time is simulated. The flight path from initial hover to engine failure is calculated as follows: For STOL operation, a linear oblique path by using the maximum takeoff power is assumed. The slope of this takeoff path is also a parameter to be optimized. For VTOL operation, ascent at a constant flight speed is assumed because VTOL takeoff requires backing up at a low speed.

These nonlinear optimal control problems are numerically solved using the sequential conjugate gradient restoration algorithm.[9] The solution technique is detailed in Ref. 7.

Category A STOL Operation

Maximum Takeoff Weight

The maximum weight for category A operation is limited by the OEI climb performance requirements[2] as shown in Fig. 2. Therefore, the maximum takeoff weight for category A STOL operation is determined independently of the takeoff path—provided the takeoff field is long enough to handle both the RTO and CTO from the CDP. Figure 3 shows the maximum weight of an existing twin-engine helicopter for category A operation as a function of density altitude. Shown are the certificated weight and the calculated maximum weight to maintain a 150 ft/min climb performance at 1000 ft above the takeoff surface when using the remaining engine's 30-min power. Some discrepancies occur between the calculation and the certification, especially at heavier weights, due to the transmission capacity for the all-engines-operating (AEO) power being disregarded in the present helicopter model.

Optimal CDP

The optimization problem for STOL operation is formulated to minimize the RTO distance (case 1a) or the CTO distance (case 1b) as listed in Table 1. The terminal condition for the RTO is that the touchdown speed factor, which is a sum of the squares of nondimensionalized forward speed and rate of descent at touchdown, is within the landing gear capacity. It is assumed that a constant distance of 100 ft from the touchdown point is required to come to a complete stop. The termination of the CTO is regulated to satisfy the following three requirements: 1) minimum clearance of 35 ft above the takeoff surface; 2) a positive rate of climb; and 3) the takeoff safety speed, i.e., the speed at which a 100 ft/min rate of climb is assured.[2] Accordingly, the minimum height of 35 ft and the minimum rate of climb of 100 ft/min are specified as the terminal conditions for the CTO. The attainment of the takeoff safety speed is not explicitly specified in the calculation.

The calculated optimal RTO path from the certificated CDP (defined as 40 kt and 40 ft) is shown in Fig. 4. A normal takeoff slope of 1/12 (corresponding to a flight path angle of 4.8 deg), given in this helicopter's flight manual, is assumed during the AEO ascent. The calculated RTO distance is about 10% shorter than the flight test result shown by the dotted line. The measured and calculated time histories are compared in Fig. 5. The present theory's results indicate that after 7 s past engine failure, the rotor speed should be maintained

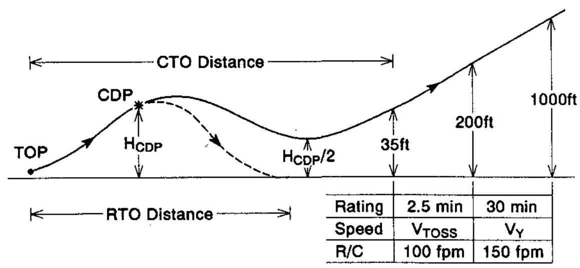

Fig. 2 OEI climb performance requirements for category A certification.

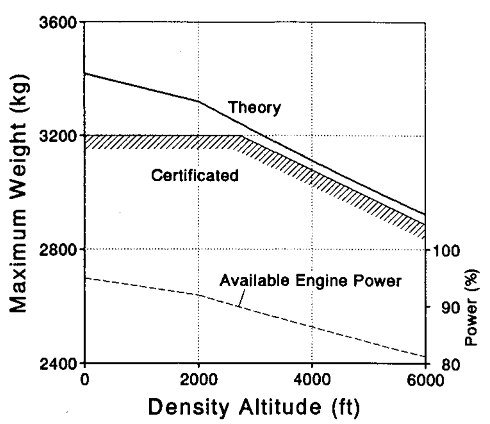

Fig. 3 Maximum weight for category A STOL operation.

Table 1 Performance index and terminal conditions

Case	Mode	Performance index	Terminal conditions
1a	STOL RTO	min $x(t_f)$	$z(t_f) = H_{CDP}$, $\Theta(t_f) = \Theta_f$ $[u(t_f)/u_{max}]^2 + [w(t_f)/w_{max}]^2 \le 1$
1b	STOL CTO	min $x(t_f)$	$z(t_f) \le H_{CDP} - 35$ ft $w(t_f) \le -100$ ft/min, $q(t_f) = 0$
2a	VTOL Weight	$\min_{H_{CHP}} \max_{\theta_0, \theta_s} m$	$x(t_f) = x_f[z(t_f)]$, $z(t_f) = H_{CHP}$, $\Theta(t_f) = \Theta_f$ $[u(t_f)/u_{max}]^2 + [w(t_f)/w_{max}]^2 \le 1$
2b	VTOL CTO	min $z(t_f)$	$w(t_f) = 0$, $q(t_f) = 0$
2c	VTOL RTO	$\min[\{u(t_f)/u_{max}\}^2 + \{w(t_f)/w_{max}\}^2]$	$z(t_f)$; given, $\Theta(t_f) = \Theta_f$

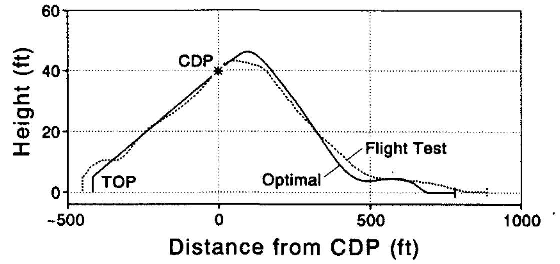

Fig. 4 Comparison of calculated and measured rejected takeoff paths (m = 3200 kg, h_d = 1400 ft).

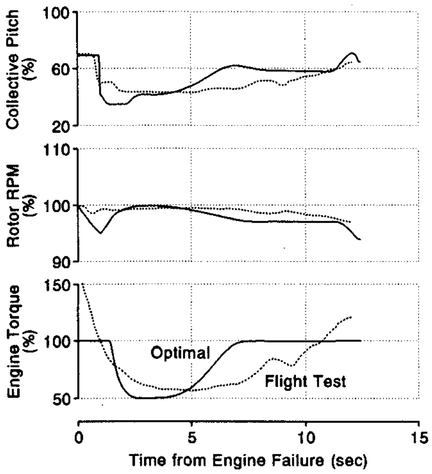

Fig. 5 Time histories of rejected takeoff from the certificated CDP (m = 3200 kg, h_d = 1400 ft).

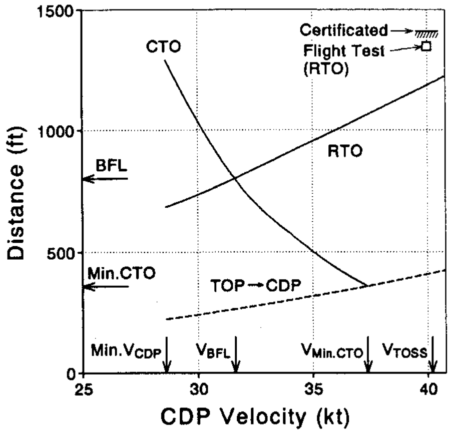

Fig. 6 Variation of continued and rejected takeoff distances vs CDP velocity (m = 3200 kg, h_d = 1400 ft).

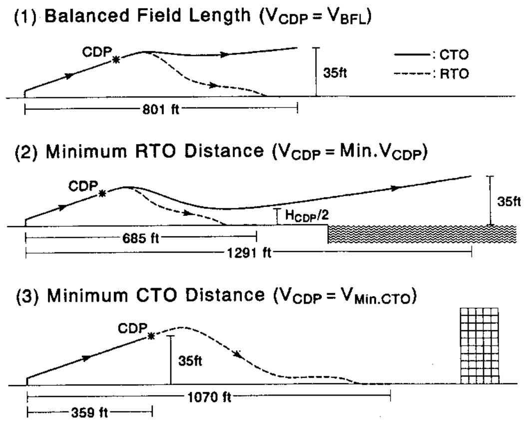

Fig. 7 Optimal CDP velocities for various heliport configurations (m = 3200 kg, h_d = 1400 ft).

within 97% rpm so that the maximum (100%) engine torque is utilized. In contrast, the test pilot used a lower collective pitch than the optimal solution, and engine torque is then reduced by the engine governor to prevent the rotor from overspeeding. This difference may cause the variation observed between the measured and calculated RTO distances.

Figure 6 shows the variation of the RTO and CTO distances with the CDP velocity for the normal takeoff slope of 1/12. Since a linear AEO takeoff path is assumed, the CDP height increases with the CDP velocity. The rectangular symbol indicates the RTO distance measured in the flight test for the certificated CDP velocity, the hatched line is the certificated required takeoff distance, and the solid lines are the calculated RTO and CTO distances, which include the AEO flight distance from the takeoff point (TOP) to the CDP indicated by the broken line. The minimum CTO distance (Min.CTO) is given by the AEO takeoff distance necessary to clear 35 ft above the takeoff surface when the CDP velocity ($V_{Min.CTO}$) is large enough to allow a CTO with no height loss. The minimum CDP velocity (Min.V_{CDP}) is limited to comply with the regulation that the maximum height loss during the CTO is within one-half the CDP height (see Fig. 2). Since the required takeoff distance is defined as the longer of the RTO and CTO distances, it is minimized in the balanced field length (BFL) condition, being nearly 30% shorter than the calculated RTO distance for the certificated CDP velocity (40 kt). The BFL condition is realized when the CDP velocity (V_{BFL}) is 8 kt lower than the calculated V_{TOSS} while the certificated CDP velocity approximates the V_{TOSS}.

The concept of the balanced field length is suitable for a fixed-wing airplane because it is accelerated on the ground, and both the RTO and CTO require the respective runway lengths. For a helicopter, the RTO alone requires a landing site and so the balanced field length does not necessarily give the minimum required heliport length. Figure 7 exemplifies the tradeoffs between the RTO and CTO distances according to the heliport configuration. When the takeoff field is clear as in a riverside heliport, the minimum CDP velocity can be applied as shown in Fig. 7 (2), with the required heliport length being nearly 15% shorter than the balanced field length shown in Fig. 7 (1). If obstacles exist in the takeoff field as shown in Fig. 7 (3), the CDP velocity should be large enough to assure the safety during the CTO.

Optimal Takeoff Path

The optimal CDP velocity depends not only on the operating conditions but also on the takeoff path. The normal takeoff slope of 1/12 is assumed in the preceding discussions because it is applicable for all operating conditions. A steep takeoff in a fully loaded condition violates the *H-V* boundary for the failure of one engine as shown in Fig. 8. When the payload is off-loaded due to the takeoff field limitations, the unsafe region in the *H-V* diagram is reduced and allows a steeper takeoff path to be applied.

Figure 9 shows the variation of the balanced field length with the takeoff slope for two different takeoff weights (3200 and 3000 kg). The hatched line indicates the maximum takeoff slope for each weight which is limited by the regulation that a minimum 5 kt clearance from the *H-V* boundary must be kept throughout the takeoff path. It should be noted that the normal takeoff slope of 1/12 is nearly optimal within the takeoff corridor for the fully loaded condition (i.e., 3200-kg weight). As for the 3000-kg weight off-loaded condition, the balanced field length is minimum at a 1/6 takeoff slope and is about 25% shorter than that of a 1/12 takeoff slope.

The variation of the RTO and CTO distances with the CDP velocity using this off-loaded condition is shown in Fig. 10. The required takeoff distance for the certificated CDP velocity of the normal takeoff slope can be reduced by about 60% when the takeoff slope and the associated CDP velocity are appropriately determined for this operating condition. It is also observed that the RTO distance is given by a nearly linear function of the CDP velocity independently of the takeoff slope. In contrast, the CTO distance is affected by the takeoff slope especially in the case of a shallow takeoff. This occurs because the shallow takeoff results in a low CDP height, thereby requiring a longer CTO distance to clear the minimum height of 35 ft.

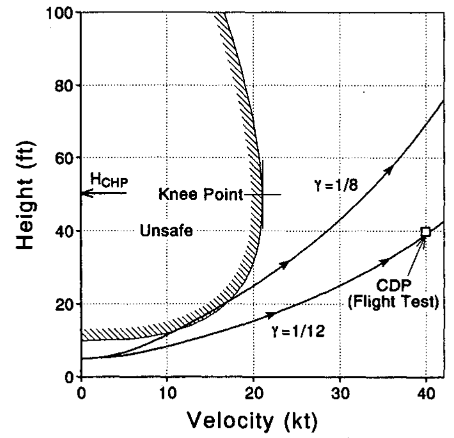

Fig. 8 Loci of takeoff trajectories on the *H-V* diagram (m = 3200 kg, h_d = 1400 ft).

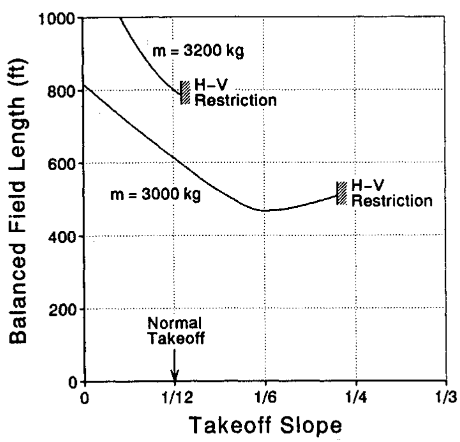

Fig. 9 Variation of balanced field length vs takeoff slope (h_d = 1400 ft).

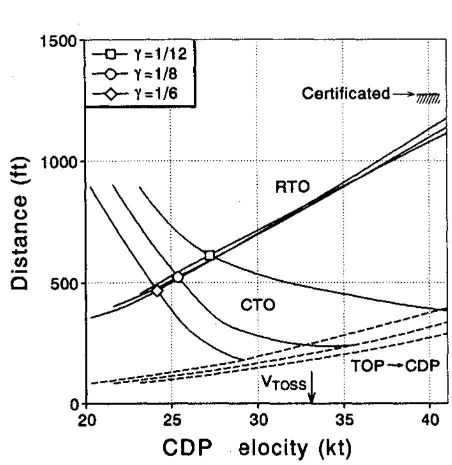

Fig. 10 Variation of continued and rejected takeoff distances vs CDP velocity (m = 3000 kg, h_d = 1400 ft).

Category A VTOL Operation

Maximum Takeoff Weight

The maximum takeoff weight in category A VTOL operation is limited by the safe landing capability for the RTO rather than by the climb performance requirement for the CTO. The optimization problem is therefore formulated to maximize the takeoff weight using the terminal condition that the touchdown speed factor following one engine failure at the critical height point (CHP) is within the safety limit (Table 1, case 2a). Here, the CHP is defined as the point where the touchdown speed factor is at a maximum, i.e., the most critical point to make a safe RTO; thus requiring the performance index of a minimax form. The CHP height H_{CHP} is found to be near the "knee point" height of the *H-V* boundary as shown in Fig. 8. Since a confined heliport is assumed in VTOL operation, the flight distance from engine failure to touchdown $x(t_f)$ is specified as a function of the engine failure height so that the helicopter returns to its inital takeoff point. The maximum takeoff weight in VTOL operation is consequently related to the location of the CHP, which in turn depends on the AEO takeoff path.

Figure 11 shows the maximum weight of the twin-engine helicopter for category A VTOL operation. The solid line indicates results of the present theory assuming the normal AEO takeoff path presented in this helicopter's flight manual, i.e., a linear backing-up path from initial hover to a prescribed CDP location. The calculated maximum weights show good agreement with the certificated weights shown by the hatched line.

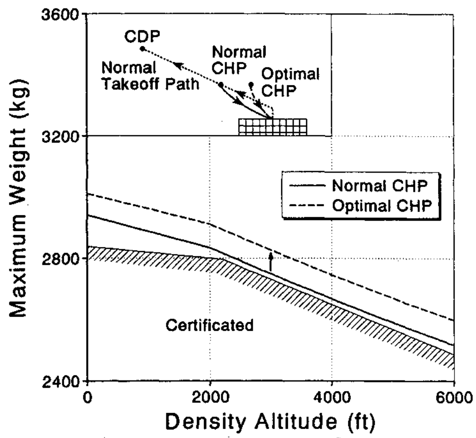

Fig. 11 Maximum weight for category A VTOL operation.

Optimal CDP

The optimal CDP in VTOL operation is theoretically determined as follows:

1) Minimum height loss during the CTO, i.e., transition from one engine failure to level flight, is calculated using the formulation listed in Table 1, case 2b.

2) The minimum height of the CDP is obtained by adding the regulatory dictated minimum clearance of 35 ft above the takeoff surface to the calculated height loss during the CTO.

3) The horizontal distance between the CDP and the heliport is determined as the resultant flight distance when the touchdown speed factor during the RTO is minimized (Table 1, case 2c).

The CDP determined in this way is shown in Fig. 12. The optimal CDP location and subsequent CTO path (solid line) are in good agreement with those of a flight test assuming the prescribed CDP location (dotted line), and hence the normal AEO takeoff path is nearly optimized with respect to the engine failure at CDP.

Optimal Takeoff Path

The optimal AEO takeoff path in VTOL operation should be determined so that the CHP location is optimized, because

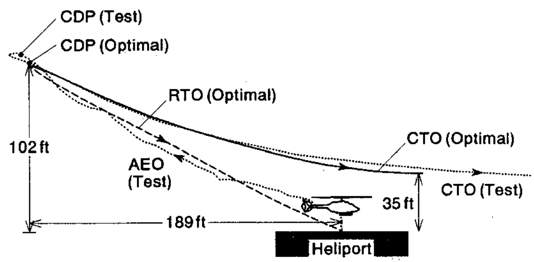

Fig. 12 Optimal CDP and continued takeoff path in comparison with flight test result (m = 2900 kg, h_d = 860 ft).

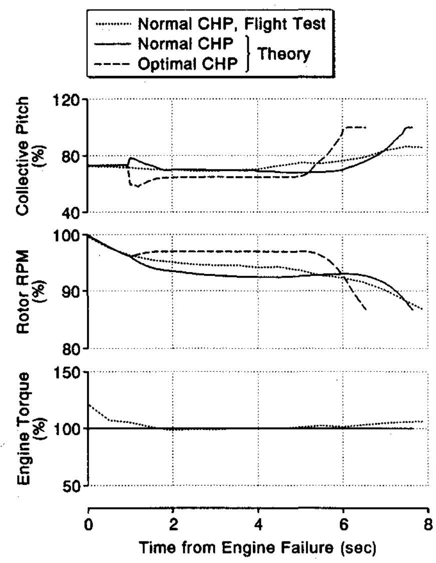

Fig. 13 Time histories of rejected takeoff from the critical height point (m = 2890 kg, h_d = 1540 ft).

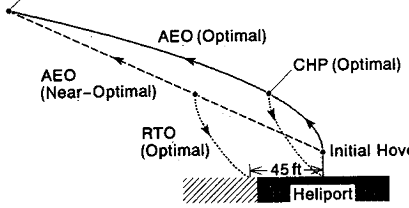

Fig. 14 Optimal takeoff paths for VTOL operation.

it is not the CDP, but instead the CHP which limits the takeoff weight. The optimal distance between the CHP and the heliport to maximize the takeoff weight is obtained as the resultant flight distance when the terminal specification of $x(t_f) = x_f[z(t_f)]$ in Table 1, case 2a is omitted. The calculated result of the optimal CHP location is also shown in Fig. 11. The broken line shows calculated maximum weights using this optimal CHP location. Note that when the AEO takeoff path is selected to pass through this optimal CHP, the theoretical maximum weight increases by 80 kg, corresponding to nearly 10% of the payload.

Figure 13 shows the time histories of the RTO from the CHP, where the flight test data for the CHP on the normal takeoff path, as well as the results of the present theory using the normal and optimal CHPs are indicated. Since the normal CHP is located farther from the heliport than the optimal CHP, a higher collective pitch is required to fly this longer distance; thus less rotor rotational energy remains for the collective flare before touchdown. This necessitates a more severe takeoff weight limitation.

Although the CHP is the point where the touchdown speed factor during the optimal RTO (Table 1, case 2c) is at its maximum, a linear takeoff path, determined from initial hover and passing through the optimal CHP, imposes so steep a landing path that it increases the touchdown speed during the RTO from the CDP due to encountering the vortex ring state,[7] as well as making it difficult for the pilot to keep the heliport in sight. The solid line in Fig. 14 shows an optimal AEO takeoff path, on which all the points including the CHP and CDP are located so that the touchdown speed factor during the optimal RTO is minimized. If such a curved takeoff path can be tracked, both the maximum safety against engine failure and maximum takeoff weight can be obtained.

It must be remembered that the above discussion is based on the condition that the helicopter comes back to its initial takeoff point during the RTO so that the required heliport size is minimized. When the heliport is long enough to allow landing from the CHP at 45 ft behind the takeoff point, the normal takeoff path, i.e., a linear backing-up path from initial hover to the optimal CDP, is also optimized with respect to the engine failure at CHP as shown in Fig. 14 by the broken line, being the most practical, near-optimal, AEO takeoff path.

Conclusions

The optimal takeoff procedures of an existing twin-engine helicopter for category A V/STOL operations are presented. It is demonstrated that by applying optimal control theory there is a possibility to expand the current operating limitations which are determined experimentally. It is also shown that the present theory is useful in reducing the cost, time, and risk of flight tests for certification.

The conclusions concerning this study are as follows:

1) The present theory can predict the maximum takeoff weight and the minimum takeoff distance for category A V/STOL operations in good agreement with flight test results.

2) The required heliport size can be significantly reduced if the takoff path and the associated CDP are specifically optimized for each of the operating conditions.

3) Optimization of the takeoff path for category A VTOL operation has the possibility of increasing the payload by 10%.

Acknowledgment

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